Subtraction Does Not Mean "Take Away"

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When subtraction is taught, it's almost always explained as a process of "taking away" some number from another number. This has been a staple of math education in the U.S. going back at least as far as textbooks from the mid 1800s.

Everybody knows that 5 - 3 means "5 take away 3"…right?

That's what I was taught.

The problem is…that's not what subtraction is.

Right now you're probably thinking one of two things…

1. That's ridiculous. It's been taught that way for so long because that's what it is.

OR

2. Ok, so what?

I know. Because I had both of these reactions at one point.

It was only when I began to look into why students have so much difficulty understanding why subtracting a negative number is the same as adding a positive number that I was forced to take this issue of "what subtraction is" seriously.

Once I did that, I started digging. What I found was both surprising and enlightening. Before I get into that, let's look at some problems that are unavoidable when we define subtraction as take away.

First, some typical application problems. Since the 1980s it's been generally accepted that there are three types of application problems based on a single operation of addition and subtraction:

- Change problems
- Combination problems
- Comparison problems

Different people may use different labels for these types but they're basically referring to the same things.

So let's see how the "take away" definition of subtraction relates to these different types of problems. For each type of problem, you can create one addition problem and two subtraction problems. Only the two subtraction ones are relevant.

Change Problem

The prototypical Change problem is a straightforward application of the take away idea. Here's an example:

1) Sue had 5 apples. She gave 2 of them to Tom. How many does she have left?

To find out how many she had left, you "take away" the 2 she gave to Tom from the 5 she had before and you see that she had 3 left.

Here's the second kind of subtraction problem for the same situation:

2) Sue had 5 apples. She gave some of them to Tom. She has 3 left. How many did she give Tom?

Again, to find out how many she gave Tom, you need subtraction. If subtraction is "taking away," you can think of this situation in one of two ways:

- You can think of it as "taking away" an unknown number (the number she gave to Tom), then realizing you need to subtract a <u>different</u> number (the number she kept) to find the unknown number, which is the number she gave away.
- You can think of this as "taking away" the 3 she had left to find the number she gave Tom. But this situation doesn't "feel" as much like a "take away" situation, does it? The 3 she had left are not the ones "taken away.". Intuitively, the ones taken away are the ones she gave to Tom.

Neither of these is a straightforward application of "taking away" because the number taken away, 2, is not the number you subtract, 3.

Combination problems

The second kind of application is a combination problem. Here's an example.

3) Sue and Tom have 5 apples in all. Sue has 3 apples. How many does Tom have?

If I'm a first grade student, am I going to intuitively see this as a "take away" situation?

Not likely.

In fact, it's hard to see how the "take away" idea has any relevance in this situation. There's no action, much less an action involving take away.

The other subtraction problem that can be created in this situation is very similar:

4) Sue and Tom have 5 apples in all. Tom has 3 apples. How many does Sue have?

Again, the idea of "taking away" is not helpful in solving this problem, even though it's a subtraction problem.

Comparison problems

The third kind of application is what's called a comparison problem. Here's an example:

5) Sue has 5 apples. Tom has 3 apples. How many more apples does Sue have than Tom?

Of course, this is a subtraction problem. The problem is that it has nothing to do with "taking away" anything. You can't "take away" Tom's apples from Sue's apples because Tom's apples are not included in Sue's apples. That's also true for the other possible subtraction problem in this situation:

6) Sue has 3 apples. Tom has 5 apples. How many more apples does Tom have than Sue?

So now we have 6 subtraction problems for the common types of application problems.

- One is clearly a "take away" situation
- One is sort of related to "take away" but not as much as the first one.
- Two involving <u>combination</u> problems have <u>nothing</u> to do with "take away"
- Two involving <u>comparison</u> problems have <u>nothing</u> to do with "take away"

In short, only one out of the possible six situations can be easily interpreted as "take away."

There's even another type of application that's not as widely noticed, called an "equalize" situation that poses the same problem:

7) Sam has 7 cards in his collection. Sue has 12 cards in her collection. How many more cards does Sam need to have as many as Sue?

This is a very common kind of scenario, and it requires subtraction to solve.

But it has nothing to do with "taking away" anything either. In fact, Sam needs to <u>acquire more</u> cards, and his cards are separate from Sue's cards. So this definitely has nothing to do with taking away cards from Sue.

So that's 7 different subtraction situations and only one truly involves "taking away" something.

Okay. So why does all this matter? Plenty.

Students need to learn to solve many types of problems including these.

But there is extensive research spanning more than 30 years that tells us that a high percentage of students have great difficulty learning to solve all these problem types. This research has shown among other things, that indeed, problem type 1 above – the one that's clearly a "take away" problem – is the <u>easiest</u> for students to learn how to solve.

But there's a flip side.

What's often overlooked is that the second type of Change problem (2) is much <u>more difficult</u> for students than the other types. It's actually one of the two <u>most difficult</u> of all the problem types associated with these three categories of problem.

Furthermore, all the "combination" (3 and 4) and "comparison" (5 and 6) problems involving subtraction are also more difficult for students than the "pure" take away problem (1).

The implication is that thinking of subtraction as "take away" may help in a few situations, but it creates difficulties in <u>all</u> the others.

These difficulties can easily be traced to the definition of subtraction as "taking away." If subtraction is taking away, a student might ask, why do you use subtraction to solve problems that have nothing to do with "taking away?"

But that's not all. The unfortunate influence of equating subtraction with "take away" goes even further.

The other most difficult type is the <u>addition</u> problem that goes with this kind of "change" problem:

8) Sue had some apples. She gave 2 of them to Tom and kept the other 3. How many did she start with?

This is actually one of the most difficult problem types for students to learn to solve. It's often referred to as a "missing start" problem, because the "starting number" is not given. Many students aren't able to do this until grade 3, and some never learn to solve it. One possible reason for this is that if you think of subtraction as "take away," you may be misled into thinking that this must be a subtraction problem, because the problem is based on "taking away" 2 apples.

But, even though it sounds like a “take away” situation, it’s not solved using subtraction. And, to make matters worse, it’s not possible to “model” the “take away” action typically used for subtraction using objects or pictures because of the unknown starting value. There’s nothing to “take away” from.

So here’s what we have. We tell the child that subtraction means “take away.”
But then, we throw them a curveball:

- Some problems that require <u>subtraction</u> to solve DON’T involve anything being “taken away.”
- Some problems that require <u>addition</u> to solve DO involve something being “taken away.”

If we think young children are not affected by such inconsistencies, we’re kidding ourselves. It’s almost certainly the major contributor to the difficulties students have learning what subtraction is and how to solve application problems using subtraction.

So what does all of this tell us?

It says to me that the “take away” interpretation of subtraction is not just wrong - it’s detrimental. It does not really reflect what subtraction is, especially when it is applied to everyday situations. The “take away” approach applies in such a small percentage of problem types that it’s bound to be more harmful than helpful.

But, of course, you may have a different take on it.